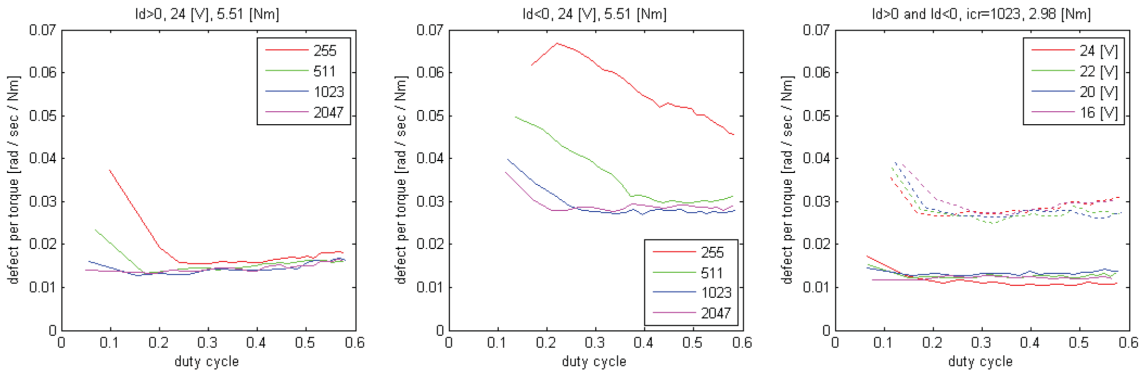


Figure 4.8: The functions $B^{+}_{\mathsf{icr}}$ (left), and $B^{-}_{\mathsf{icr}}$ (middle) for different modulation frequencies of $31.25, 15.63, 7.81$, and $3.91$ kHz. Right: The functions $B^{\pm}_{1023}$ obtained from the experiment at different supply voltages.

## 4.4 Position and velocity control

In order to vary the inclination of the joints of Marsokhod, we implement position control with bounded velocity. We utilize the same model to accelerate the forward drive and the turning. The theory is presented in

**Lemma 4.15.** With a coefficient $\mu > 0$ and for a constant $c$, we investigate the differential equations

$$\partial_t p(t) = v(t) \tag{4.8}$$

$$\partial_t v(t) = \mu(c - v(t)) \tag{4.9}$$

Using Laplace[1]-transformation and integration, we obtain the solution as

$$p(t) = p_0 + v_0\frac{1}{\mu}(1 - e^{-t\mu}) - c\frac{1}{\mu}(1 - t\mu - e^{-t\mu}) \tag{4.10}$$

$$v(t) = v_0 e^{-t\mu} + c(1 - e^{-t\mu}) \tag{4.11}$$

where $(v_0, p_0)$ represents the instantaneous state $(v(0), p(0))$. We bound the parameter $|c| \le C$, and assume $|v_0| \le C$. According to (4.9), these bounds limit $|v| \le C$, and $|v'| \le 2\mu C$.

We are interested in the choice of parameter $c$ with $|c| \le C$ to reach a target state $(v_{\mathrm{d}}, p_{\mathrm{d}})$ from the instantaneous state $(v_0, p_0)$ in least time. The time $T$ to attain $v(T) = v_{\mathrm{d}}$ with $c = C \operatorname{sgn}(v_{\mathrm{d}} - v_0)$ is $T = \frac{1}{\mu}\log\frac{c - v_0}{c - v_{\mathrm{d}}}$. We evaluate

$$p(T) = p_0 + \frac{1}{\mu}\left(v_0 - v_{\mathrm{d}} + c\log\frac{c - v_0}{c - v_{\mathrm{d}}}\right)$$

which does not necessarily equal $p_{\mathrm{d}}$. For convenience, we define $p_T = p(T)$. Time-optimal control is achieved with $c = C \operatorname{sgn}(p_{\mathrm{d}} - p_T)$. $\square$

[1] Pierre Simon Laplace, * 23. Mar 1749 in Beaumont-en-Auge, † 5. Mar 1827 in Paris

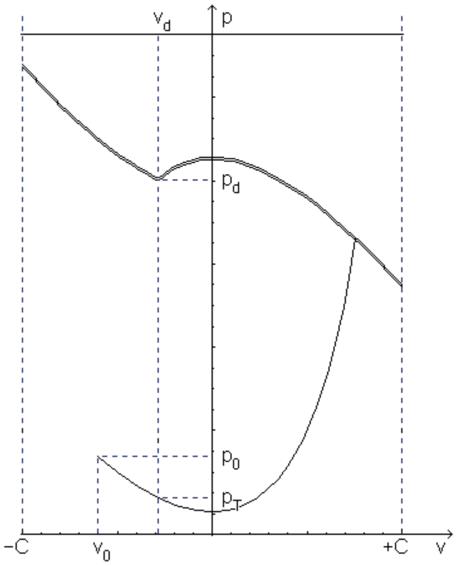


Figure 4.9: Phase plot to Lemma 4.15. The sign of the control $c = \pm C$ at time $t = 0$ depends on whether $(v_0, p_0)$ lies above or underneath the bold line that contains $(v_\mathrm{d}, p_\mathrm{d})$.

To apply the theory in the lemma, we discretize the functions (4.10) and (4.11) as

$$p^{k+1} = p^k + v^k \frac{1}{\mu}(1 - e^{-\mathbf{h}\mu}) - c^k \frac{1}{\mu}(1 - \mathbf{h}\mu - e^{-\mathbf{h}\mu}) \tag{4.12}$$

$$v^{k+1} = v^k e^{-\mathbf{h}\mu} + c^k (1 - e^{-\mathbf{h}\mu}) \tag{4.13}$$

where $\mathbf{h}$ is the period of one communication cycle. The parameter $c^k \in [-C, C]$ is selected based on the type of control:

*Velocity control:* To reach and maintain the destination velocity $v_\mathrm{d}$, we set

$$c^k = \mathcal{C}(v_\mathrm{d}) := \operatorname{clip}\big(-C, \frac{v_\mathrm{d} - (1 - \mathbf{h}\mu)v^k}{\mathbf{h}\mu}, C\big) \tag{4.14}$$

where $\operatorname{clip}(a, b, c) := \min(\max(a, b), c)$.

*Position and velocity control:* To attain the state $(v_\mathrm{d}, p_\mathrm{d})$, we set

$$c^k = \mathcal{C}(v_\mathrm{d}, p_\mathrm{d}) := C \operatorname{sgn}\big(p_\mathrm{d} - p^k - \frac{1}{\mu}(v^k - v_\mathrm{d} + c \log \frac{c - v^k}{c - v_\mathrm{d}})\big) \tag{4.15}$$

where $c = C \operatorname{sgn}(v_\mathrm{d} - v^k)$.

*Velocity control with bounded position:* To reach and maintain the destination velocity $v_\mathrm{d}$ with the constraint $p_{\min} \le p \le p_{\max}$, we set

$$c^k = \mathcal{C}(v_\mathrm{d}, p_{\min}, p_{\max}) := \operatorname{clip}\big(\mathcal{C}(0, p_{\min}), \mathcal{C}(v_\mathrm{d}), \mathcal{C}(0, p_{\max})\big) \tag{4.16}$$

Because of $\mathcal{C}(v_\mathrm{d}) = \mathcal{C}(v_\mathrm{d}, -\infty, \infty)$, equation (4.14) is contained in (4.16). In case $v_\mathrm{d} = 0$, the control (4.15) is a special case of (4.16), since $\mathcal{C}(0, p_\mathrm{d}) = \mathcal{C}(0, p_\mathrm{d}, p_\mathrm{d})$.

**Example 4.16.** Figure 4.10 demonstrates the three types of control. □

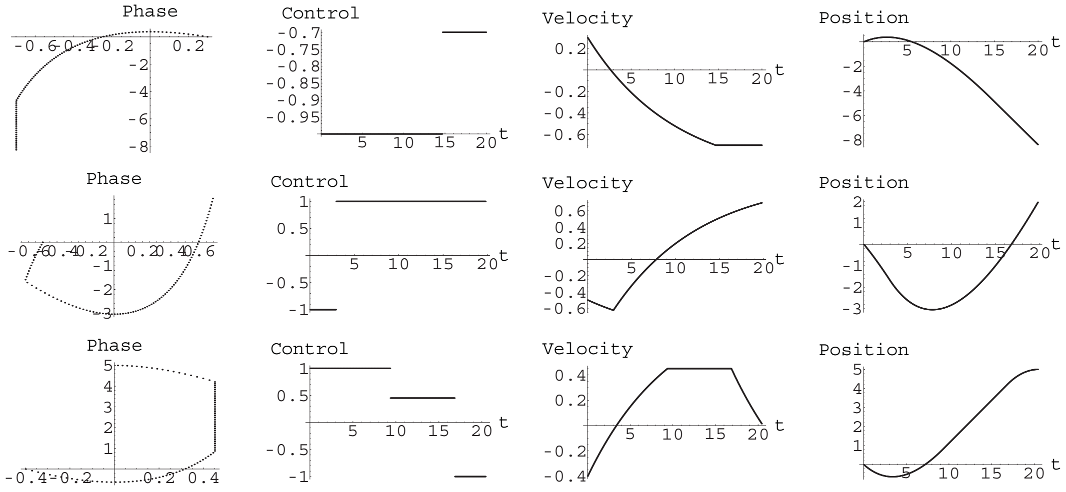


Figure 4.10: Illustration of the three types of control with $\mu = 0.1$, $C = 1$, and $p_0 = 0$. Top: Velocity control with $v_0 = 0.3$, and $v_\mathrm{d} = -0.7$. Middle: Position and velocity control $v_0 = -0.5$, and $(v_\mathrm{d}, p_\mathrm{d}) = (0.7, 2)$. Bottom: Velocity control with bounded position $v_0 = -0.4$, $v_\mathrm{d} = 0.45$, while $p_\mathrm{min} = -5$, and $p_\mathrm{max} = 5$.

To control the inclination of the front joint of Marsokhod, the motor is assigned a velocity $\hat{w}_6(t)$ of the form (4.11). According to the equation, the function $\hat{w}_6(t)$ is continous. Using the conversion (4.3), the velocity $\hat{w}_6(t)$ translates into a continous duty cycle

$$d_6(t) := W_{\mathsf{icr}}^{-1}\left(\frac{\hat{w}_6(t)}{\zeta_6}\frac{\mathbf{U}}{U}\right)$$

Since the slope of the duty cycle $d_6(t)$ is limited, we guarantee low mechanical stress on the motor as well as on the chassis of Marsokhod. We test the accuracy of the position control in

**Procedure 4.17.** Marsokhod is located on flat terrain. We assign a sequence of target positions $p_6^\mathrm{d}$ to be attained by the front joint

$$p_6^\mathrm{d} = -2.21 \;\rightarrow\; -2.51 \;\rightarrow\; -1.61 \;\rightarrow\; -1.81 \;\rightarrow\; -1.21$$

while $v_6^\mathrm{d} = 0$. The transition to the next element in the sequence occurs when $|p_6 - p_6^\mathrm{d}| \leq 0.05$ [rad] during a period of 1 s. Then, the joint has attained the target position up to a certain accuracy and has stopped moving. At the beginning, the joint is in vertical position $p_6 = -1.2121$ rad, recalling that $\gamma_f = -p_6$. The initial position of the motor is $\bar{p}_6 = -1.2121 + b_6$. Figure 4.11 evaluates the positions $\bar{p}_6, p_6$ and the velocities $\hat{w}_6, w_6$.

Analogous, we assign a sequence of target positions $p_7^\mathrm{d}$ to be attained by the rear joint

$$p_7^\mathrm{d} = 2.21 \;\rightarrow\; 2.51 \;\rightarrow\; 1.61 \;\rightarrow\; 1.81 \;\rightarrow\; 1.21$$

At the beginning, the joint is in vertical position $p_7 = 1.2121$ rad, recalling that $\gamma_r = p_7$. The initial position of the motor is $\bar{p}_7 = 1.2121$ rad. Figure 4.12 evaluates the positions $\bar{p}_7, p_7$ and the velocities $\hat{w}_7, w_7$. □

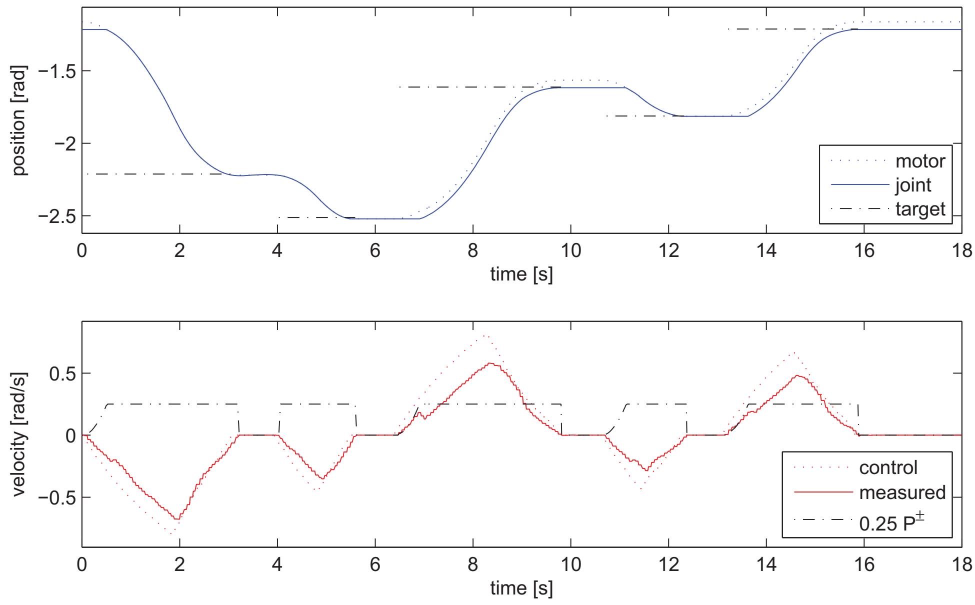


Figure 4.11: Position control of the front joint of Marsokhod using (4.15) with the parameters $\mu = 0.4$, $C = \frac{1}{2}\mathbf{w}$. The position of the motor $\bar{p}_6$ and the inclination of the joint $p_6$ deviate at most by the backlash of $b_6 = 0.052$ [rad]. The lower frame evaluates $P^{\operatorname{sgn} w_6}(\bar{p}_6, p_6, b_6)$, which is not a velocity but a scalar.

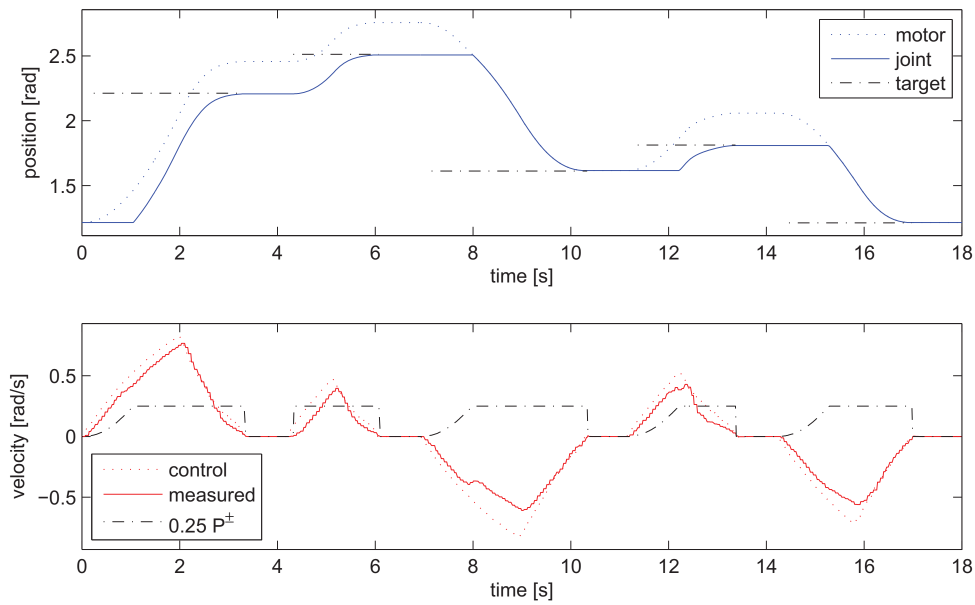


Figure 4.12: Position control of the rear joint of Marsokhod. The position of the motor $\bar{p}_7$ and the inclination of the joint $p_7$ deviate at most by the backlash of $b_7 = 0.250$ rad. The lower frame evaluates the scalar $P^{\operatorname{sgn} w_7}(\bar{p}_7, p_7, b_7)$.